

Exponential Functions

November 29, 2010

Investigate

There is a connection between population growth, radioactive decay, musical scales, and compound interest. They seem to have little in common, but you can model any of them using an exponential function.

An exponential function has the general form $f(x) = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.

Properties of Exponential Functions

Q1. The graph of $f(x) = 2^x$ is shown. What is the domain of $f(x)$? What is the range of $f(x)$?

Q2. Change the values of parameters a and b by dragging them. Observe the resulting changes in the graph. Does the domain and range change?

Q3. Hit the show "y-intercept" button. How does this value relate to $y = a \cdot b^x$?

(Hint: drag slider a).

Q4. What are the x and y -intercepts of the graph? Explain how the intercepts are related to parameters a and b .

Q5. In the general form of the exponential function, there are three constraints ($a \neq 0, b > 0$ and $b \neq 1$). Explain the reason for each of these constraints. Answering each part below may help you figure out how to answer this question.

(a) What happens to the function $f(x)$ when $a > 1$?

(b) What happens when $0 < a < 1$?

(c) What happens to $f(x)$ when a is negative?

(d) How does a change the domain and range of $f(x)$?

Q6. What happens to $f(x)$ when $b > 1$? When $0 < b < 1$?

A function $f(x) = ab^x$ is considered to be exponential growth when $b \geq 1$, and decay when $0 < b < 1$.

Q7. Write equations for the following parameters for a and b . State the domain, range and y -intercept of each function. Tell whether the function represents growth or decay.

(a) $a = 3, b = 5$

(b) $a = -2, b = \frac{1}{2}$

(c) $a = 4, b = 2$

Q8. Using what we know about transforming functions, take an educated guess on how sliders h and k effect $f(x)$. Move sliders h and k , do the movements of h and k differ from you guess? Explain.

Q9. Describe the transformations below with respect to $f(x) = a \cdot (b)^x$, then graph each function. State the domain of each function and whether or not the function is exponential growth or decay.

(a) $f(x) = 3 \cdot 2^x + 1$

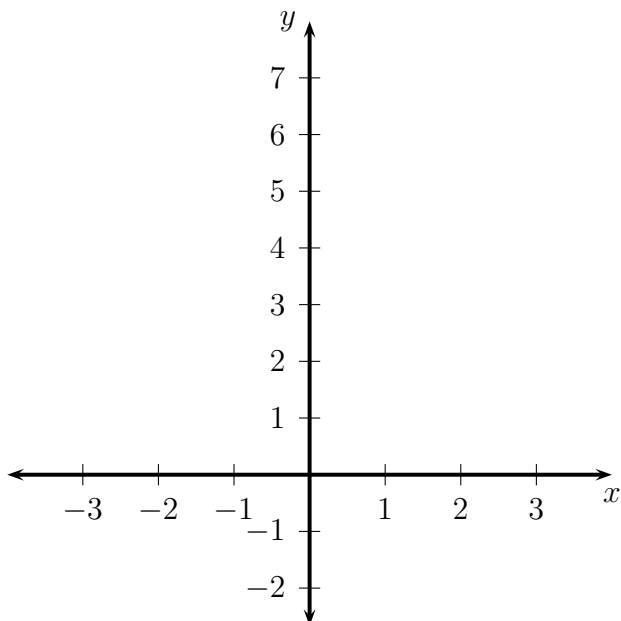
(b) $g(x) = -2 \cdot \left(\frac{1}{2}\right)^x$

(c) $h(x) = 2^{x-3} - 4$

Graphing

1. Hit the reset button and THEN select the **Show Points** button.

Q10. Create a table of values of the points on $f(x)$. Sketch the function below (note: the axes aren't equal).



Q11. What do you notice about negative x -values and their output? Is this always the case then $a > 1$?

2. Consider the parent function $f(x) = 3^x$.

Q12. Use a table of values to plot the following:

(a) $g(x) = 3^{x+2} - 1$

(b) $h(x) = 3^{x-4} + 2$

(c) $q(x) = -3^{x+2} - 1$

